① If $\int_1^a (3x^2+2x+1)\,dx = 11$, find real values of a.

Sol:

we have $\int_1^a (3x^2+2x+1)\,dx = 11$

$\Rightarrow \left(\frac{\cancel{3}}{\cancel{3}}x^3 + \frac{\cancel{2}}{\cancel{2}}x^2 + x\right)_1^a = 11$

$\Rightarrow \left(x^3+x^2+x\right)_1^a = 11$

$\Rightarrow (a^3+a^2+a) - (1+1+1) = 11$

$\Rightarrow a^3+a^2+a-3 = 11$

$\Rightarrow a^3+a^2+a-14 = 0.$

$\Rightarrow (a-2)(a^2+3a+7) = 0.$

$$\begin{array}{c|cccc} \therefore 2 & 1 & 1 & 1 & -14 \\ & 0 & 2 & 6 & 14 \\ \hline & 1 & 3 & 7 & \boxed{0} \end{array}$$

$\Rightarrow a = 2.$ $(\because a^2+3a+7 \neq 0$ for $a \in R)$

② If $\int_a^b x^3\,dx = 0$ and if $\int_a^b x^2\,dx = \frac{2}{3}$, find a and b.

Sol:

we have $\int_a^b x^3\,dx = 0$

$\Rightarrow \left(\frac{x^4}{4}\right)_a^b = 0$

$\Rightarrow \left(\frac{b^4}{4} - \frac{a^4}{4}\right) = 0.$

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$\Rightarrow \frac{b^4 - a^4}{4} = 0$

$\Rightarrow b^4 - a^4 = 0$.

$\Rightarrow (b^2 - a^2)(b^2 + a^2) = 0$.

$\Rightarrow b^2 - a^2 = 0 \Rightarrow (b+a)(b-a) = 0$.

$\Rightarrow \quad b = -a \quad (\because b \neq a)$

$\therefore \boxed{b = -a}$

Now, $\int_a^b x^2 dx = \frac{2}{3}$

$\Rightarrow \left(\frac{x^3}{3}\right)_a^b = \frac{2}{3}$

$\Rightarrow \frac{b^3}{3} - \frac{a^3}{3} = \frac{2}{3}$

$\Rightarrow \frac{1}{3}(b^3 - a^3) = \frac{2}{3}$

$\Rightarrow b^3 - a^3 = 2$

$\Rightarrow (-a)^3 - a^3 = 2$

$\Rightarrow -a^3 - a^3 = 2$

$\Rightarrow -2a^3 = 2$

$\Rightarrow a^3 = -1 \Rightarrow \boxed{a = -1}$

$\therefore a = -1$ and $b = 1$

(3) Evaluate $\int_0^{\pi/2} \sqrt{1-\cos 2x}\, dx$.

Sol:

Let $I = \int_0^{\pi/2} \sqrt{1-\cos 2x}\, dx$.

$\Rightarrow I = \int_0^{\pi/2} \sqrt{2\sin^2 x}\, dx$

$\Rightarrow I = \sqrt{2} \int_0^{\pi/2} \sin x\, dx$.

$= \sqrt{2} \left[-\cos x\right]_0^{\pi/2}$

$= \sqrt{2}\left[-\cos\frac{\pi}{2} - (-\cos 0)\right]$

$= \sqrt{2}\left[-0+1\right]$

$= \sqrt{2}$.

(4) Evaluate

(i) $\int_0^{\pi/4} \tan^2 x\, dx$ (ii) $\int_0^{\pi/2} \sin^2 x\, dx$ (iii) $\int_0^{\pi/4} \sin 3x . \sin 2x\, dx$.

Sol:

(i) Let $\int_0^{\pi/4} \tan^2 x\, dx$.

$I = \int_0^{\pi/4} (\sec^2 x - 1)\, dx = (\tan x - x)_0^{\pi/4}$

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$$= \left(\tan\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\right) - \left(\tan(0) - 0\right)$$

$$= 1 - \frac{\pi}{4} - 0$$

$$= 1 - \pi/4.$$

(ii) Let $I = \int_0^{\pi/2} \sin^2 x \, dx$

$$= \int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right) dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (1-\cos 2x)\, dx$$

$$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right)_0^{\pi/2}$$

$$\left[\begin{array}{l} D(\cos 2x) = -2\sin 2x \\ \int \cos 2x \, dx = \frac{\sin 2x}{2} \end{array}\right]$$

$$= \frac{1}{2}\left(\left(\frac{\pi}{2} - \frac{1}{2}\sin \cancel{2}\left(\frac{\pi}{\cancel{2}}\right)\right) - \left(0 - \sin(0)\right)\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0 - 0 - 0\right)$$

$$= \frac{\pi}{4}$$

(iii) $\int_0^{\pi/4} \sin 3x \cdot \sin 2x \, dx$

$$I = \int_0^{\pi/4} \frac{1}{2}(2\sin 3x \sin 2x) dx$$

$$= \frac{1}{2}\int_0^{\pi/4} \left[\cos(3x-2x) - \cos(3x+2x)\right] dx.$$

$$\left[2\sin A \sin B = \cancel{\sin}\ \cos(A-B) - \cos(A+B)\right]$$

$$= \frac{1}{2}\int_0^{\pi/4} (\cos x - \cos 5x) dx$$

$$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right]_0^{\pi/4}$$

$$= \frac{1}{2}\left[\left(\sin(\pi/4) - \frac{1}{5}\sin(5\pi/4)\right) - \left(\sin(0) - \sin(0)\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{\sqrt{2}} + \frac{1}{5}\frac{(1)}{\sqrt{2}} - 0 - 0\right]$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{5\sqrt{2}}\right)$$

$$= \frac{1}{2}\left(\frac{\cancel{6}^{3}}{5\sqrt{2}}\right) = \frac{3}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{10}.$$

⑤ Evaluate

(i) $\int_0^{\pi} \sin^3 x\, dx$ (ii) $\int_0^{\pi/2} \cos^3 x\, dx$.

Sol:

(i) Let $I = \int_0^{\pi} \sin^3 x\, dx$.

$$I = \int_0^{\pi} \left(\frac{3\sin x - \sin 3x}{4}\right) dx \quad [\because \sin 3x = 3\sin x - 4\sin^3 x]$$

$$= \frac{1}{4}\int_0^{\pi} (3\sin x - \sin 3x)\, dx$$

$$= \frac{1}{4}\left[-3\cos x - \left(-\frac{\cos 3x}{3}\right)\right]_0^{\pi}$$

$$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right]_0^{\pi}$$

$$= \frac{1}{4}\left[\left(-3\cos\pi + \frac{\cos 3\pi}{3}\right) - \left(-3\cos(0) + \frac{1}{3}\cos(0)\right)\right]$$

$$= \frac{1}{4}\left[\left(-3(-1) + \frac{1}{3}(-1)\right) - \left(-3(1) + \frac{1}{3}(1)\right)\right]$$

$$= \frac{1}{4}\left(3 - \frac{1}{3} + 3 - \frac{1}{3}\right) = \frac{1}{4}\left(6 - \frac{2}{3}\right) = \frac{1}{\cancel{4}}\left(\frac{\cancel{16}^{\,4}}{3}\right) = \frac{4}{3}$$

(ii) Let $I = \int_0^{\pi/2} \cos^3 x \, dx$

$$= \int_0^{\pi/2} \left(\frac{\cos 3x + 3\cos x}{4} \right) dx \quad \left[\because \cos 3x = 4\cos^3 x - 3\cos x \right].$$

$$= \frac{1}{4} \int_0^{\pi/2} (\cos 3x + 3\cos x) \, dx$$

$$= \frac{1}{4} \left[\frac{\sin 3x}{3} + 3\sin x \right]_0^{\pi/2}$$

$$= \frac{1}{4} \left[\left(\frac{1}{3} \sin 3\left(\frac{\pi}{2}\right) + 3\sin\frac{\pi}{2} \right) - \left(\frac{\sin(0)}{3} + 3\sin(0) \right) \right]$$

$$= \frac{1}{4} \left[-\frac{1}{3} + 3 - 0 + 0 \right]$$

$$= \frac{1}{\cancel{4}} \left(\frac{\cancel{8}^{2}}{3} \right) = \frac{2}{3}$$